Earliest Arrival Flows with Multiple Sinks

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Earliest arrival flows:

- motivated by evacuation problems

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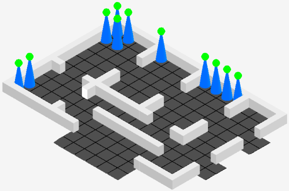

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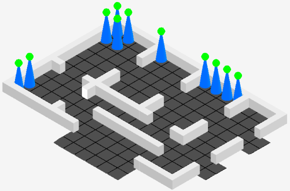

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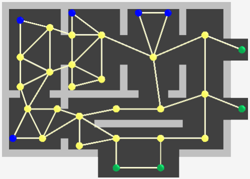

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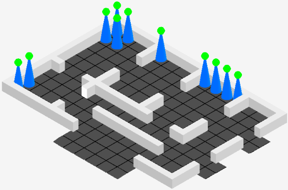
Earliest arrival flows:

- motivated by evacuation problems
- flows over time



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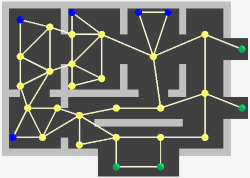
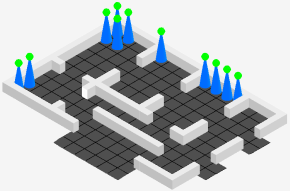


Earliest Arrival Flows with Multiple Sinks

Earliest arrival flows:

- motivated by evacuation problems
- flows over time
- usually several terminals ⇝ transshipments



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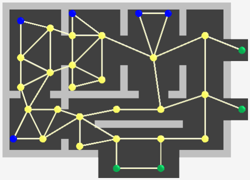
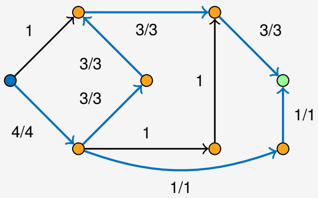


Maximum Flows



A flow in N is a mapping

$f : E \to \mathbb{R}_{\geq 0}$ that fulfills:

- $f(e) \leq u(e) \ \forall e \in E$
- $\sum_{e \in \delta^+(v)} f(e) = \sum_{e \in \delta^-(v)} f(e)$

Flow value: $|f| = \sum_{e \in \delta^-(t)} |f(e)|$

- Network $N = (V, E)$
- Capacities $u : E \to \mathbb{N}$
- Sources $s^1, \ldots$, Sinks $t^1, \ldots$

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Flows over time

add transit times $\tau : E \to \mathbb{N}$
see capacities as flow rates

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Flow value at time T:
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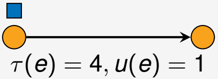

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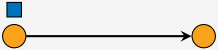

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$\tau(e) = 4, u(e) = 1$

Maximum value at $t = 5$:

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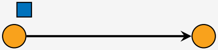

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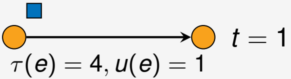

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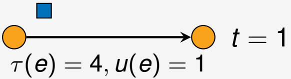

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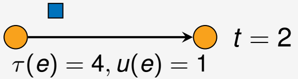

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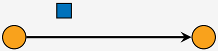

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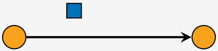

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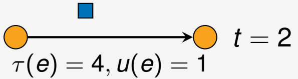

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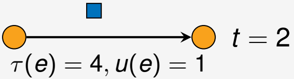

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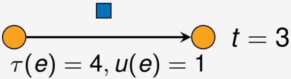

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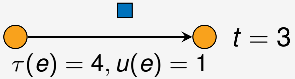

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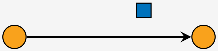

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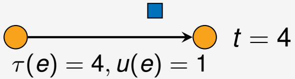

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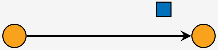

$t = 4$

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Flows over time

add transit times $\tau : E \to \mathbb{N}$
see capacities as flow rates

Maximum Flows over time

A flow over time is given by

$f : E \times \mathbb{Z}_{\geq 0} \to \mathbb{R}_{\geq 0}$ that fulfills:

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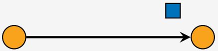

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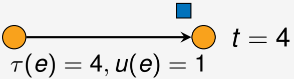

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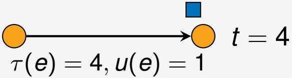

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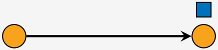

$t = 5$

$\tau(e) = 4, u(e) = 1$

Maximum value at $t = 5$: 1

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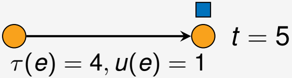

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Transshipments

- Multiple sources and sinks (terminals)
- Supplies and demands on the terminals

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Dynamic Transshipment (Transshipment over time)

- Test: Can supplies/demands be satisfied until time T?

$\Rightarrow$ Such a T is called feasible.

- Calculate transshipment.

Transshipments

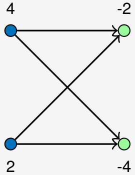
- Multiple sources and sinks (terminals)
- Supplies and demands on the terminals

Quickest Transshipment

- Calculate smallest T that is feasible.
- Calculate appropriate transshipment.

$\rightsquigarrow$ connected to evacuations

Example with zero transit times

Graph	$t = 1$	$t = 2$	$t = 3$
			

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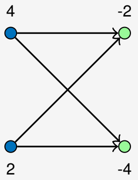	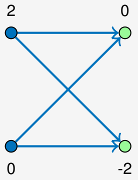	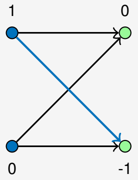	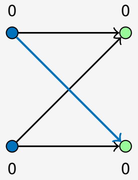

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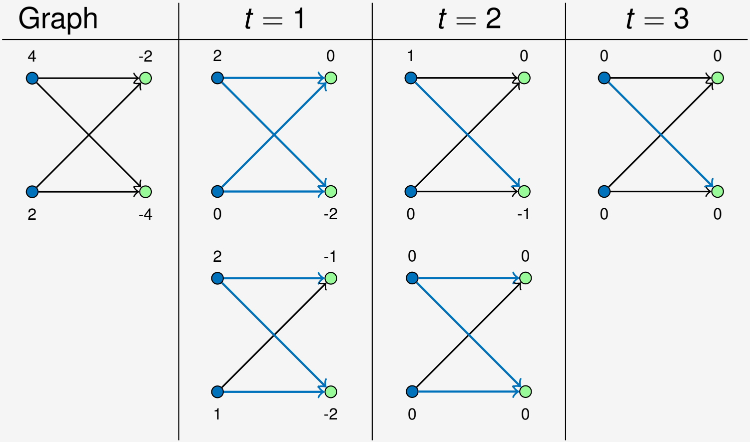


Earliest Arrival Transshipment

Simultaneously maximize flow at any point in time.

Let S^- be the set of sinks. Set

$$p(t) = \sum_{v \in S^-} \sum_{i=1}^{t} \text{(amount of flow reaching sink } v \text{ at time } i)$$

$\Rightarrow$ Simultaneously maximize $p(t)$ for all $t \in \mathbb{Z}_{\geq 0}$.

Does such a transshipment exist (in general) ?

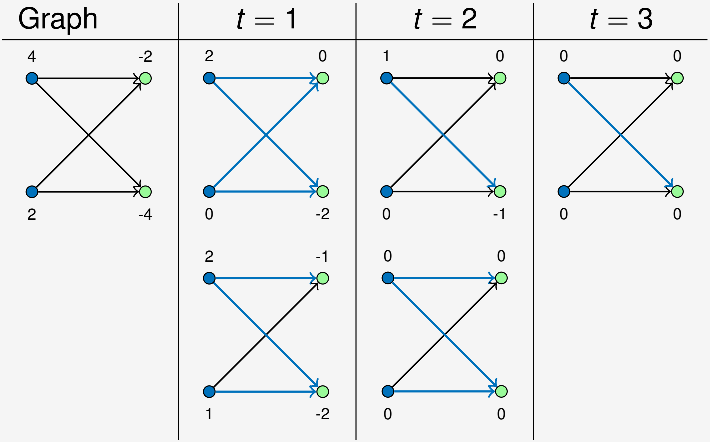
Theorem (Minieka)

In networks with only one sink, earliest arrival transshipments do always exist.

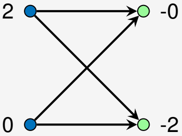
$\rightsquigarrow$ A lot of research has been devoted to single-sink-networks:

- Wilkinson, W. L., *An algorithm for universal maximal dynamic flows in a network*, Operations Research 19 (1971), pp. 1602-1612.
- Fleischer, L. K., *Faster algorithms for the quickest transshipment problem*, SIAM Journal on Optimization 12 (2001), pp. 18-35.
- Baumann, N. and M. Skutella, *Solving evacuation problems efficiently: Earliest arrival flows with multiple sources*, Mathematics of Operations Research 34 (2009), pp. 499-512.

Example with zero transit times



An Observation



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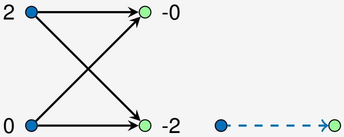
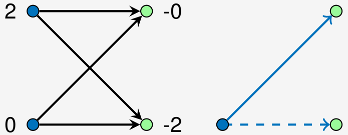


- unit cannot be sent

What if a network does not allow for Earliest Arrival Flows?

An Observation



- unit cannot be sent
- another unit must be responsible

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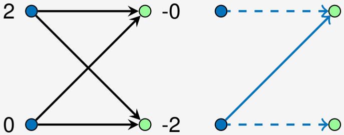

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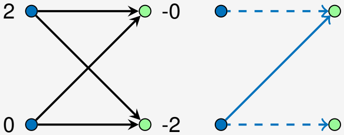

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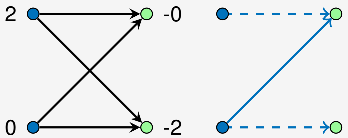

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- 2-Approximation?

Approximation Types

Let $p(t)$ be the max. flow value that can be sent up to time t.

- Value-Approximation: Up to each point in time t, send at least $p(t)/\beta$ units of flow
- Time-Approximation: Up to each point in time t, send at least $p\left(\frac{t}{\alpha}\right)$ units of flow

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Time-Approximation

- has been used before: Baumann/Köhler (2007), Fleischer/Skutella (2007), Groß/Skutella (2012)
- design of FPTASs instead of pseudopol. algorithms

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Let N be a dynamic network (with possibly multiple sources and sinks). Then there exists a 2-value-approximative earliest arrival flow in N in the discrete time model.

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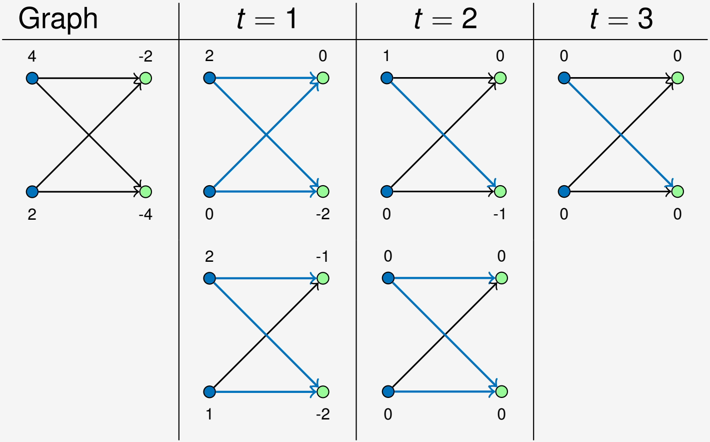
2-Approximation

- Compute maximum flow with time horizen 1
- Repeat for $i = 2, \ldots, T$:
- Compute maximum flow up to time i without changing the flow values at time $1, \ldots, i-1$

Example with zero transit times



Towards 2-Value-Approximation: Time-expanded network

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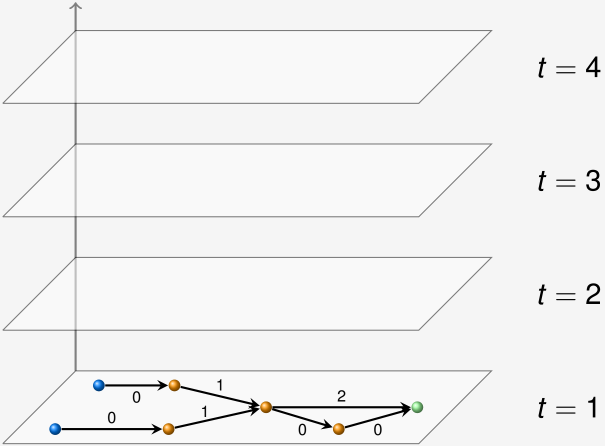

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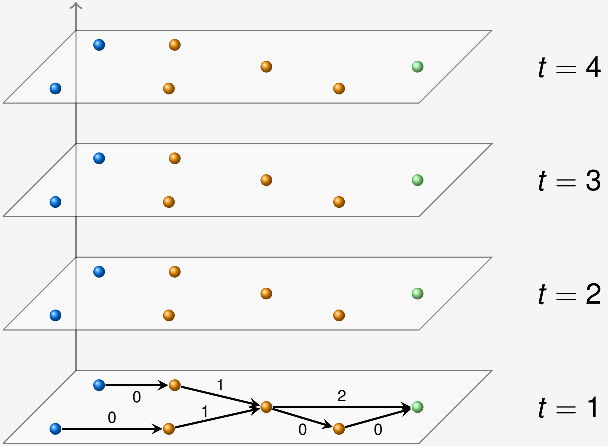

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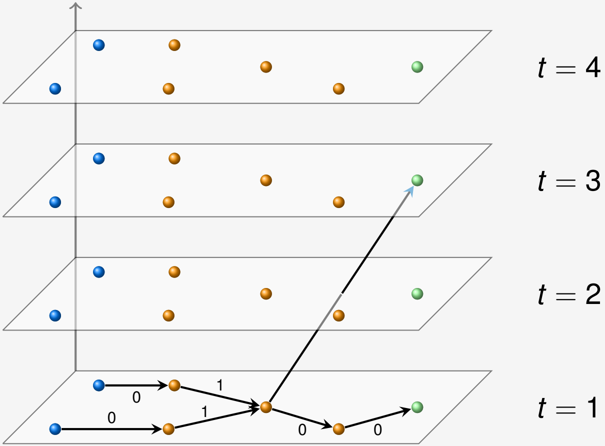

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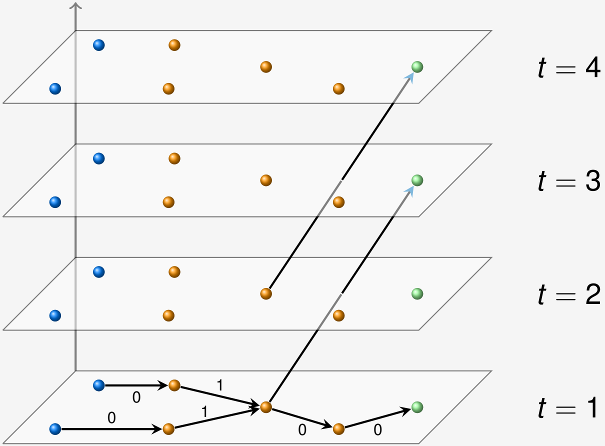

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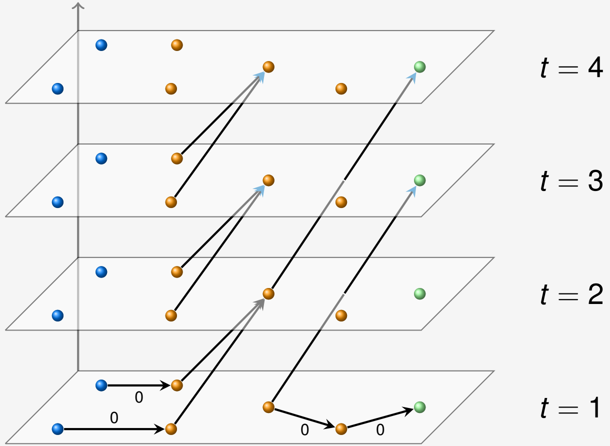

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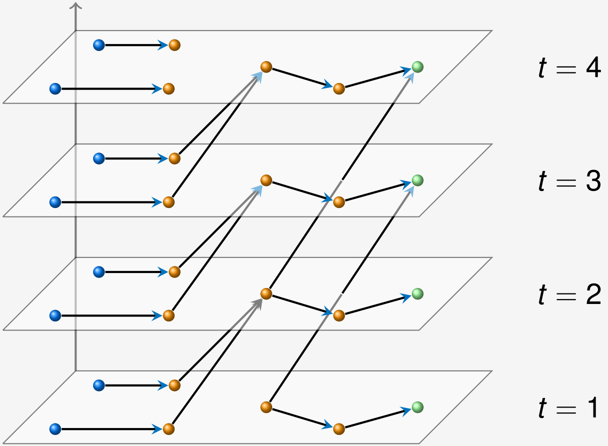

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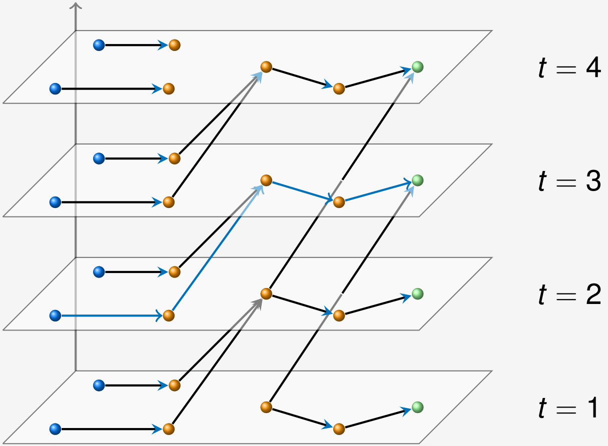

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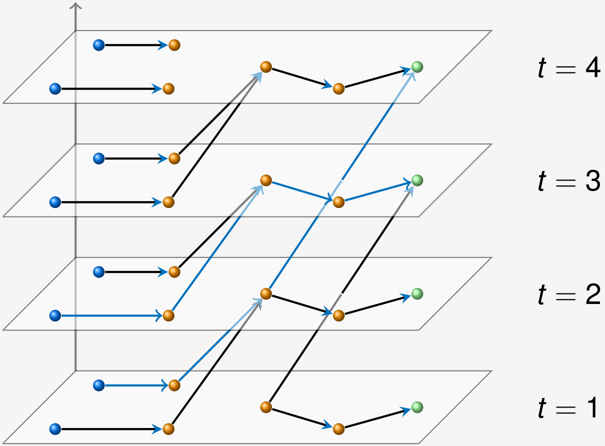

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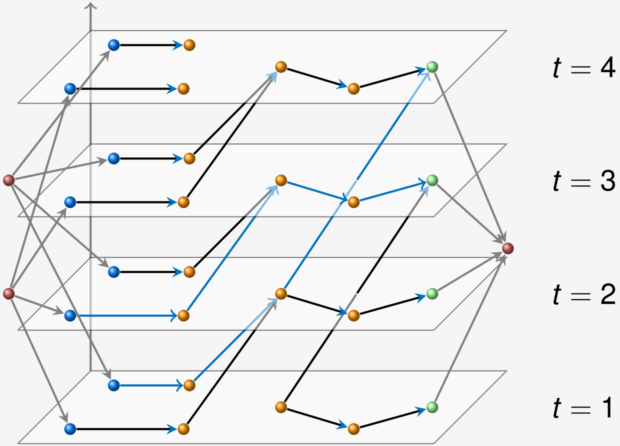

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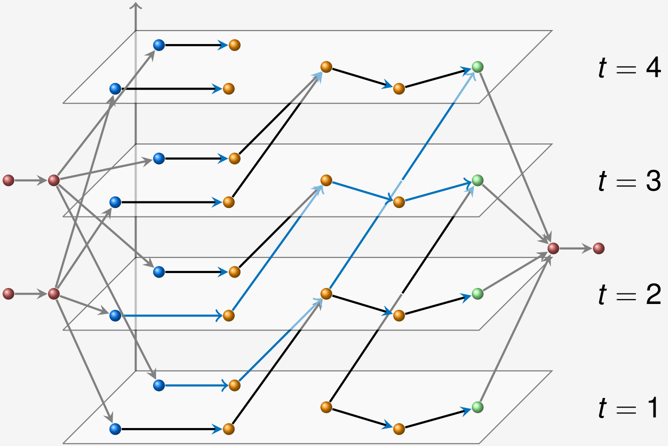

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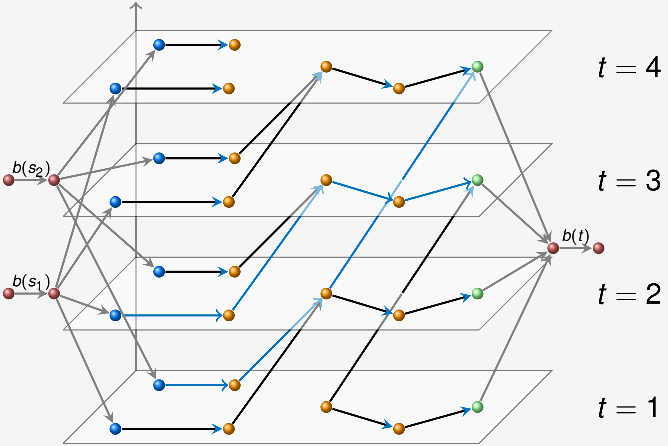

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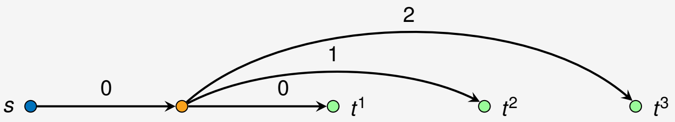

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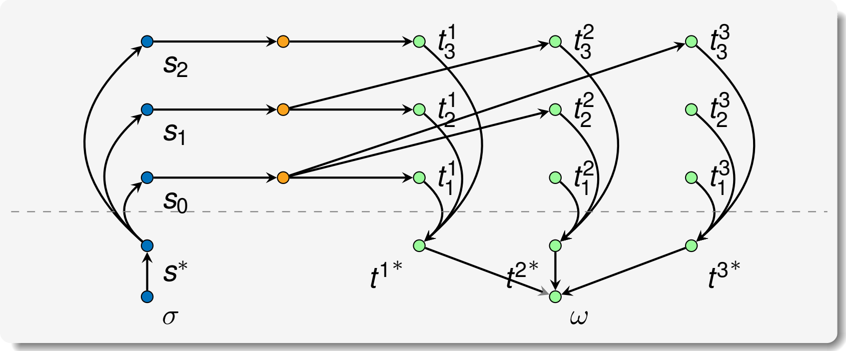

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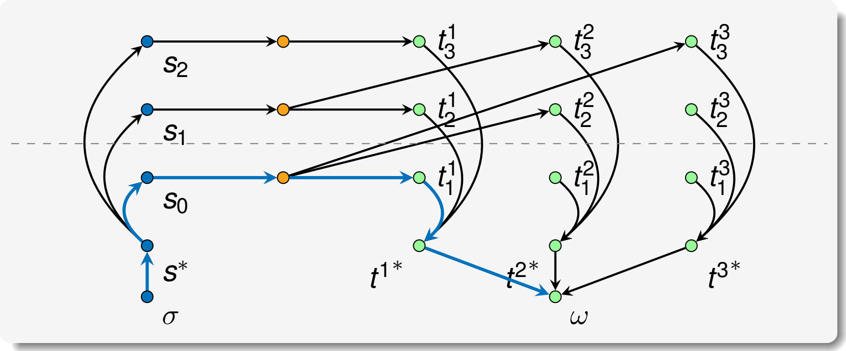

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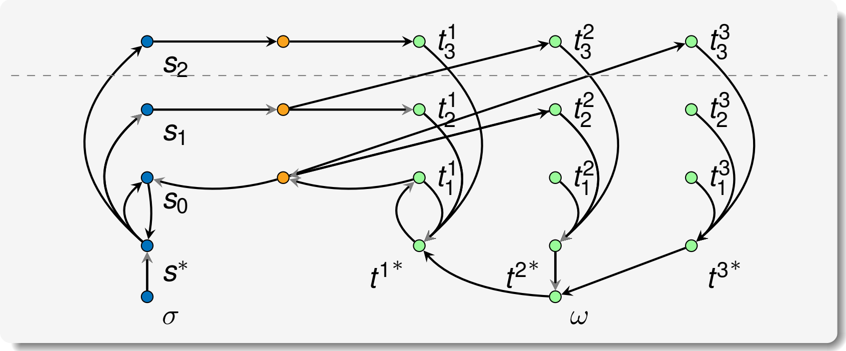

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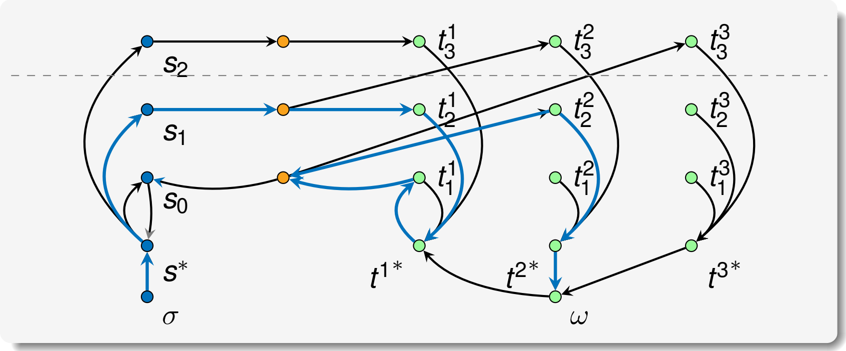

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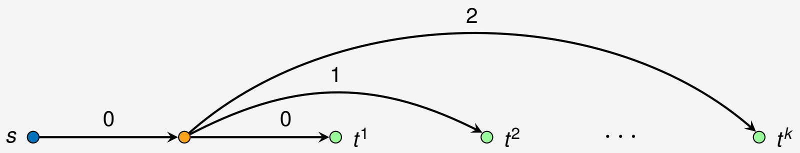
Lower bound: $\alpha, \beta \geq 2$



Known Results: Before

Sources	Sinks	$\tau \in \mathbb{R}^E$	$\tau \equiv 0$
1	1	$\alpha = \beta = 1$	
2+	1	$\alpha = \beta = 1$	
1	2+	$\alpha > 1$	$\alpha = 1$
		$\beta > 1$	$\beta = 1$
2+	2+	$\alpha > 1$	
		$\beta > 1$	

Known Results: Before and now

Sources	Sinks	$\tau \in \mathbb{R}^E$	$\tau \equiv 0$
1	1	$\alpha = \beta = 1$	$\alpha = \beta = 1$
2+	1	$\alpha = \beta = 1$	$\alpha = \beta = 1$
1	2+	$\mathbf{2} \leq \alpha \leq \mathbf{4}$ $\beta = \mathbf{2}$	$\alpha = 1$ $\beta = 1$
2+	2+	$\alpha = \boldsymbol{T}$ $\beta = \mathbf{2}$	$\alpha = \boldsymbol{T}$ $\beta = \mathbf{2}$

Theorem

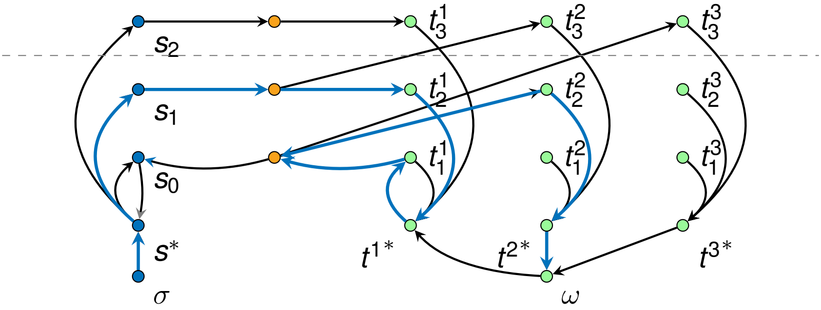
If

- $\varepsilon > 0$ is given with $1/\varepsilon$ integral
- α/β are the best possible time/value-approximation factors

then we can compute a

- $(1 + O(\varepsilon))\alpha$-time- and $(1 + \varepsilon)$-value-approximate EAF and
- $(1 + O(\varepsilon))$-time- and $(1 + \varepsilon)\beta$-value-approximate EAF

in running time polynomial in the input size and ε^{-1}.
This holds in the continuous and discrete time model.



Thank you for your attention :-)